

FAST HYPERVOLUME SAMPLING ALGORITHM

Goal: Multiobjective Optimization

In the last decades, there has been a growing interest in developing evolutionary algorithms for multiobjective optimization problems. Many variants proposed in the last years make use of special indicator functions that explicitly define the optimization goal—independent from the algorithm itself. The hypervolume indicator, first introduced by Zitzler et al. as the ‘size of the space covered’, has proven to be highly useful for search. This is mainly due to the following feature: whenever one Pareto-set approximation completely dominates another approximation, the hypervolume of the former will be greater than the hypervolume of the latter.

Unfortunately, the calculation of the hypervolume measure is computationally very demanding, and current algorithms are exponential in the number of objectives.

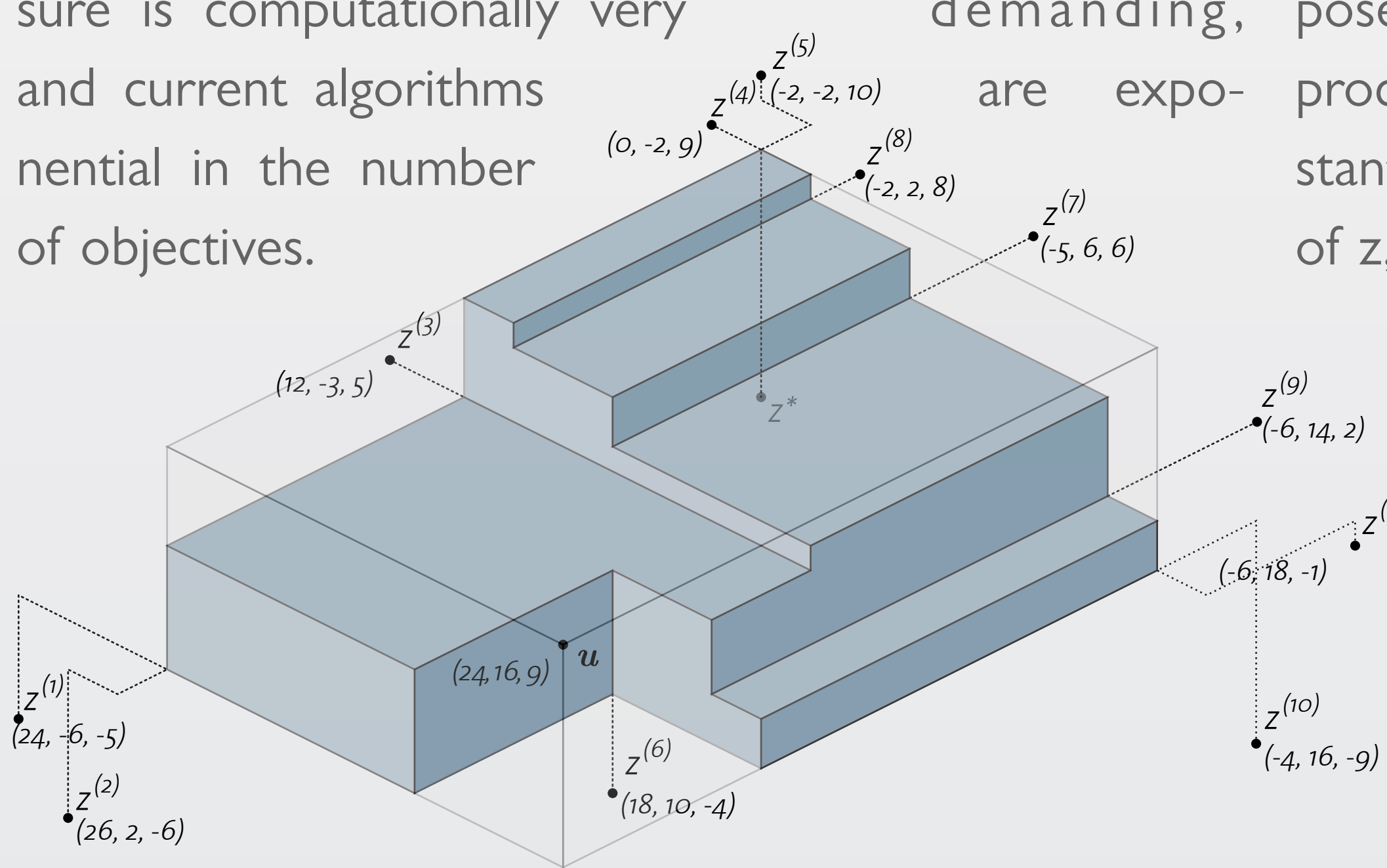


Figure 1: The shaded area shows the contribution of point z^* . By drawing samples from the sampling rectangles, this contribution can be approximated.

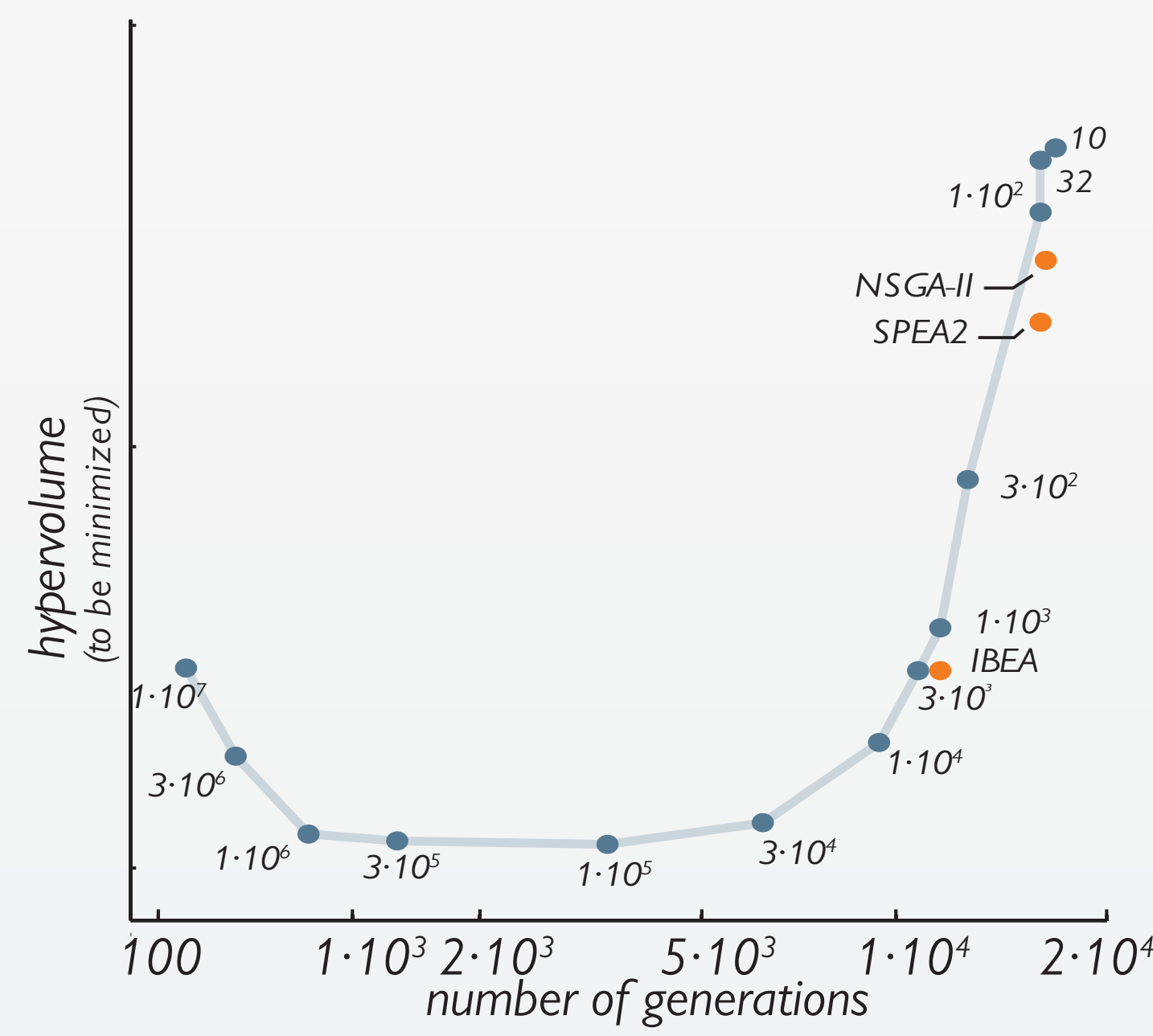


Figure 2: Performance of an evolutionary algorithm using the sampling strategy in comparison with NSGA-II, SPEA2 and IBEA

Approach: Monte-Carlo Sampling

Instead of calculating the hypervolume measure, it can be approximated by Monte-Carlo sampling. We propose a technique designed to be used in the selection process of an evolutionary algorithm which allows substantial speedups. In order to sample the contribution of z , three steps are necessary:

1. A sampling space has to be defined, which is as tight as possible (Figure 1)
2. A number of samples is drawn to estimate the contribution of each point
3. Statistical tests are performed to determine the number of samples needed to obtain a reliable decision

To determine the probability, that the contribution of a individual a is smaller than the contribution of individual b , one can use the confidence intervals proposed by Agresti and Coull:

$$P(\lambda(C_a) < \lambda(C_b)) \approx \Phi \left(\frac{\hat{\lambda}(C_b) - \hat{\lambda}(C_a)}{\sqrt{\lambda(S_a)^2 \hat{p}_a(1-\hat{p}_a) / m_a + 2 + \lambda(S_b)^2 \hat{p}_b(1-\hat{p}_b) / m_b + 2}} \right)$$

Experimental Results

Figure 2 shows a comparison of state of the art algorithms to an evolutionary algorithm based on the novel sampling strategy. The number of samples has to be chosen carefully: If the number is too small, the accuracy of environmental selection suffers and the algorithm does not converge well. On the other hand, if too many samples are used, the number of generations that can be evaluated given a constant time budget is too small. The latter problem affects the adaptive strategy to a lesser extent, since the desired accuracy is reached mostly before the number of samples exceeds its limit. The best number of samples is about 10,000 samples.

Reference

J. Bader, K. Deb, and E. Zitzler. *Faster Hypervolume-based Search using Monte Carlo Sampling*. In Conference on Multiple Criteria Decision Making (MCDM 2008). Springer 2008

APPLICATIONS - PLACING WIRELESS SENSOR NODES

Introduction: Wireless Sensor Nodes

Wireless Sensor Nodes (WSN) are a new form of pervasive and distributed computing infrastructure, deeply embedded into the environment. Providing remote access to the sensing devices, WSN technology is a radical innovation for many diverse application areas such as environmental monitoring, structural monitoring, or event detection. Monitoring phenomena in a given environment requires coverage of the area with the sensing devices.

Problem Statement: Placing Sensor Nodes

How many wireless sensor nodes should be used and where should they be placed in order to cover a certain area with as few nodes as possible but still providing reliable communication paths from each node to a data sink? This is a difficult question to answer for a decision maker due to the conflicting objectives of deployment costs (number of nodes used) and wireless transmission reliability:

$f_1 = \text{number of nodes}$

$$f_2 = \frac{1}{W} \sum_{j=1}^{N_{red}} w_j \cdot (1 - p_{worst,j}) \quad \text{with } W = \sum_{j=1}^{N_{red}} w_j$$

Approach: Using an evolutionary algorithm

Here, we address this problem using a multiobjective evolutionary algorithm (MOEA) which allows to identify the trade-offs between low-cost and highly reliable deployments. The algorithm finds a set of good solutions, from which the decision maker can choose from. To this end, various components are designed:

- Two objective functions based on a WSN deployment model.
- A representation of the WSN network, allowing varying number of wireless sensor nodes.
- A crossover operator that combines two WSN deployments
- A mutation operator based on Voronoi diagrams.

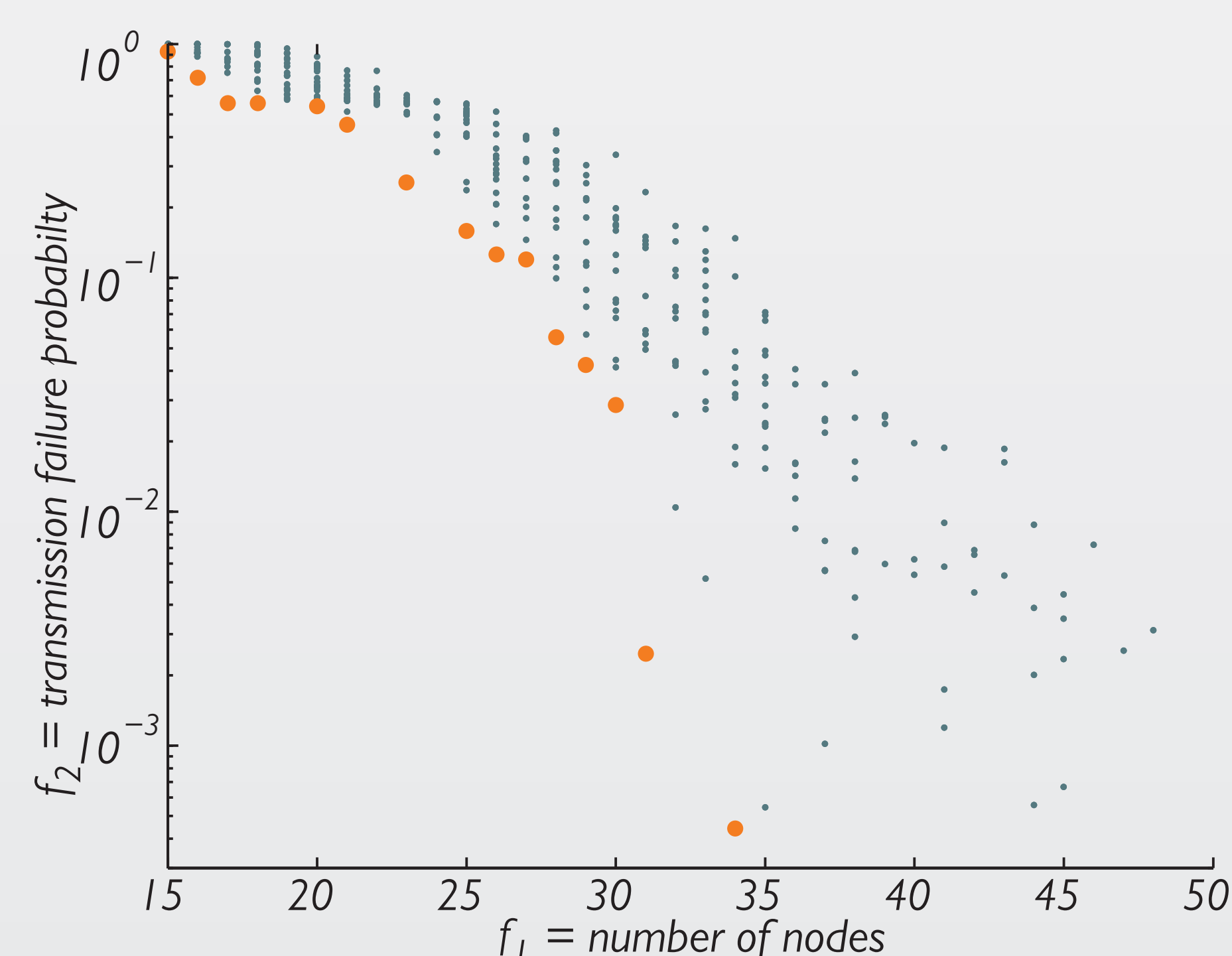


Figure 1: The number of sensor nodes deployed and the reliability of transmission form two conflicting goals

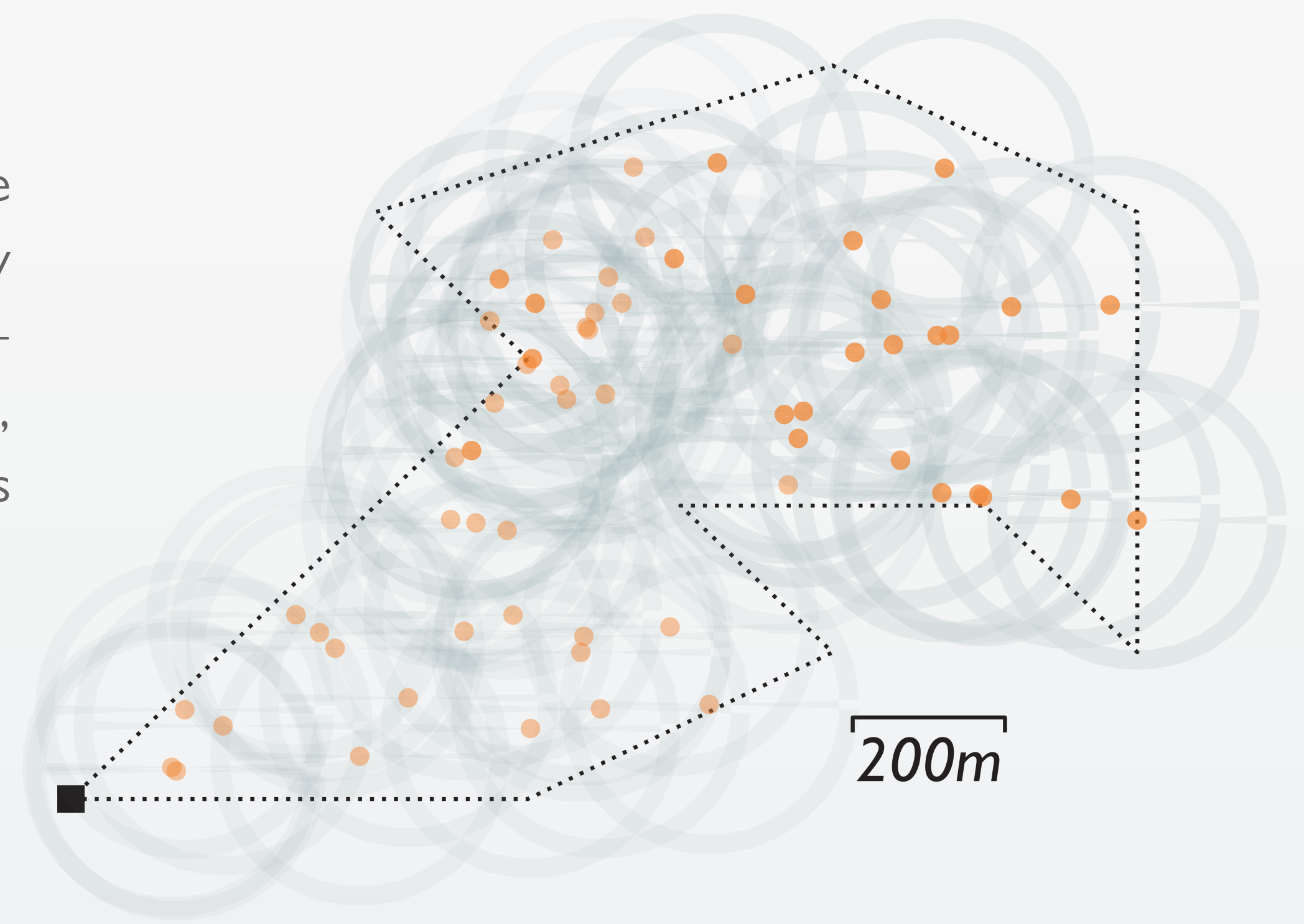


Figure 2: One deployment of 60 sensor nodes that monitor an outdoor scenario, found by the evolutionary algorithm.

Results

As Figure 1 shows, the MOEA finds a set of different solutions, using between 15 and 50 nodes and achieving different transmission error rates. The orange points correspond to the Pareto-optimal solutions. Figure 2 shows one WSN network found for a different deployment scenario.

Reference

M. Woehrle, D. Brockhoff, T. Hohm, and S. Bleuler. *Investigating Coverage and Connectivity Trade-offs in Wireless Sensor Networks: The Benefits of MOEAs*. In Conference on Multiple Criteria Decision Making (MCDM 2008). Springer